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B.A./B.Sc. (Part-III) Examination, 2019

MATHEMATICS

Paper - I

(Real Analysis)

Time Allowed : Three Hours] *[Maximum Marks : 75*

Note : Attempt **all** sections as per instructions.

Section-A

Note : Attempt **all** questions. Each question carries **2½** marks.

1. (i) Find l.u.b and g.l.b for the set

$$S = \left[\frac{1}{n} : n \in N\right]$$

(ii) State Bolzano Weierstrass theorem.

(iii) Define countable set.

(iv) Define uniform convergence.

(v) State the necessary condition of maxima and minima for function of two variable.

(vi) Define Riemann Integral.

(vii) Define Metric space.

P.T.O.

(viii) Find the limit point of the sequence

$\langle(-1)^n\rangle$

(ix) Define limit point of the set.

(x) Discuss the convergence of integral

$\int_1^{\infty} \frac{dx}{x^{3/2}}$

Section-B

Note : Attempt **all** questions. Each question carries **6** marks.

2. Prove that the sequence $\left\{\frac{2^n}{n!}\right\}$ is a monotonic decreasing sequence; also prove that it is bounded.

OR

A sequence is convergent if and only if it is Cauchy sequence.

3. Show that the sequence $\{fn\}$ where $f_n(x) = \frac{x}{(1+nx^2)}$ converges uniformly on IR

OR

Test the series $\sum \frac{\sin nx}{n^p}$ for uniform convergence in any interval.

4. Find the maximum or minimum values of the function $x^3y^2(1-x-y)$.

OR

Show that the function

$$f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^6}, & x \neq 0, \ y \neq 0 \\ 0, & x = 0, \ y = 0 \end{cases}$$

is not continuous at (0,0) in (x,y).

OR

5. Prove that every continuous function is R-integrable.

OR

Show that the function f(x)=sinx is integrable on $\left[0, \frac{\pi}{2}\right]$

6. The set Z, of integers with the usual metric $(d(x,y) = |x-y| \forall x, y \in Z)$ is a complete metric space.

OR

If A and B are closed sets of a metric space (X,d) then show that

(a) $A \cup B$ is closed

(b) $A \cap B$ is closed

Section-C

Note : Attempt any **two** questions. Each question carries **10** marks.

7. State and Prove Darboux theorem.

8. Test the convergence of following integrals :

(i) $\int_1^{\infty} \frac{dx}{\sqrt{x^5+1}}$

(ii) $\int_0^{\infty} \frac{x^3 dx}{(x^2+a^2)^2}$

9. Expand $\tan\theta$ in powers of $x - \frac{\pi}{4}$ using Taylor's Theorem.

10. Test for uniform convergence of the series

$$\sum x \left(\frac{n}{1+n^2x^2} - \frac{(n+1)}{1+(n+1)^2x^2} \right)$$

11. Prove using definition of L(P,f) and U(P,f)

$$\int_1^2 (3x+1)\, dx = \frac{11}{2}$$